Hanoi Revisited
Thomas E. Prescott and Janet L. Gruber
Denver Veterans Administration Medical Center, Denver, Colorado

Mary Olson
Griffiths-Lerabbee, Inc., Gardiner, Maine

Karlene C. Fuller
Denver Veterans Administration Medical Center, Denver, Colorado

Three years ago Prescott, Loverso, and Selinger (1984) described some differences between normal and left-brain-damaged (aphasic) subjects' performance on a nonverbal problem solving task. In that paper those authors raised questions regarding problem solving deficits in aphasia and of the nature of cognitive deficits in aphasia. That study compared patients with aphasia to non-brain-damaged subjects relative to each group's abilities on a nonverbal problem solving task called the Towers of Hanoi problem. Weisenberg and McBride (1935) demonstrated that aphasic subjects showed poorer performance on nonverbal tests than a normal control group did. In 1964, Weinstein discussed aphasic patient deficits in "organization and selection of materials" regardless of the nature of the task. Bygotsky (1962) argued that language has become so closely intertwined with thought that any language deficit would necessarily affect cognition. In 1966, Luria concluded from a problem-solving experiment that some of his aphasic patients had difficulty grasping logical as well as grammatical relationships.

Problem solving has been described as an overt or covert behavioral process that generates potentially effective solutions to an identified problem. The process then, increases the possibility of choosing the most effective alternative from the available others (D'Zurrilla and Goldfried, 1971). There are four component behaviors involved in the problem solving process: (1) understanding the problem, (2) planning a solution, (3) carrying out the plan, and (4) checking and modifying the results (based on feedback).

Little research has been done linking problem solving abilities and aphasia. Smith (1980) was concerned with examining nonverbal inferential abilities compared with nonverbal memory in aphasic subjects. Subjects were asked either to infer size of objects or to make judgments of size based on previously learned information. Smith concluded that intellectual deficits in aphasia result from problems in short-term memory and not logical thinking. It is difficult, however, to separate problem solving deficits from memory deficits in this study (Prescott, Loverso, and Selinger, 1984, p. 235).

Prescott, Loverso and Selinger (1984) reported on a comparison of 18 nonaphasic (normal) subjects with 18 aphasic (left brain damaged) subjects specific to their problem solving ability on the Towers of Hanoi puzzle. Their results indicated poorer performance for the aphasic group than for the normal group on this task. Considerably more moves were required by the brain-damaged aphasic group to solve the problem and over one-half (56%) failed to solve the problem within the maximum number of moves required by normals (11) to solve the problem.
METHOD

The purpose of this investigation was to determine whether or not patients with left, right and bilateral brain damage, respectively, differed from non-brain-damaged (normal) subjects and from each other on the Hanoi Towers puzzle, Raven's Coloured Progressive Matrices (1968), the Columbia Mental Maturity Scale (1972) or the Porch Index of Communicative Ability (1981).

Subjects. Four groups of subjects were selected for this study. The 10 normal subjects had no reported history of brain damage. Ten subjects were included in the left-brain-damaged group, 10 in the right-brain-damaged group and 10 in the bilaterally-brain-damaged group. All brain-damaged subjects had experienced only one episode of brain damage as reported in their medical records and by case history interview.

Procedure. All subjects were asked to complete 4 experimental tasks. The order of presentation for each task was randomized for each subject. Each subject was asked to complete the Towers of Hanoi puzzle, the Columbia Mental Maturity Scale (CMMS), Raven's Coloured Progressive Matrices (RCPM) and the Porch Index of Communicative Ability (PICA). The Columbia Mental Maturity Scale, Raven's Coloured Progressive Matrices and the Porch Index of Communicative Ability were presented as directed by the test protocols. For the Towers of Hanoi puzzle, subjects were seated at a table with the puzzle before them. The Towers of Hanoi puzzle (Figure 1) consists of three disks which must be moved from peg A to peg C. Only one disk may be moved at a time, and no disk may be placed on top of a disk smaller than itself. Each move of a disk by a subject was recorded by the examiner. Seven moves are the fewest required to complete the puzzle. Solution of the problem required a degree of planning in selecting moves and generating subgoals that would bring the problem closer to solution. Both pictured and verbal instructions were presented to the patients. The experimenter repeated the instructions until each subject had no questions regarding the rules of the task. Following instruction, patient responses were counted and timed with a stopwatch.

![Towers of Hanoi puzzle](image)  
Figure 1. Towers of Hanoi problem.

RESULTS AND DISCUSSION

An analysis of variance for repeated measures yielded a statistically significant difference (p < .001) between the four groups on PIICA performance. These data are presented graphically in Figure 2. Tukey's test for a significant gap (1963) indicated that the normal group was significantly superior
(p < .05) to all other groups (X = 14.36). All brain-damaged groups were significantly different (p < .05) from each other (X = 10.16, 13.20, and 11.84 for the left-brain-damaged, right-brain-damaged, and bilaterally-brain-damaged groups respectively). Performance on the PICA separated the subject groups, with normals exhibiting the highest performance level followed by right-brain-damaged, bilaterally-brain-damaged and left-brain-damaged subjects respectively.

Figure 2. Mean PICA scores for normal, left, right, and bilaterally brain-damaged subjects.

Discussion following the paper presented in 1984 by Prescott, Loverso, and Selinger suggested the need for study of subject groups beyond the normal and left-brain-damaged groups as well as for the study of performance on other problem solving tasks. The Columbia Mental Maturity Scale and Raven's Coloured Progressive Matrices were selected for this study as measures that would add information to some of the discussion questions raised at that time. The CMMS is a problem-solving task requiring a subject to select an item that doesn't fit from a group of 3 to 5 related items.

A repeated measures analysis of variance on CMMS performance yielded significant differences (p < .04) among the groups. Post hoc testing (Tukey's test for significant gap, 1963) indicated significant differences (p < .05) between the normal group and the three pathologic groups. No significant differences (p > .05) were noted between the three pathologic groups (Figure 3). These data are graphically represented in Figure 3.

According to Raven (1968),

The coloured Matrices... can be used satisfactorily with people who, for any reason, cannot understand or speak the English language, with people suffering from physical disabilities, aphasics,... as well as with people who are intellectually sub normal or have deterioriated. The scale as a whole can be described as "a test of observation and decreased thinking." (p. 3)
The totals for Levels A, B, and AB of Raven's Coloured Progressive Matrices (Figure 4) were also determined for all subjects. A repeated measures analysis of variance yielded no statistically significant difference ($p > .05$) between any of the subject groups on this measure. Normal subjects performed better than any other group although the differences were not statistically significant.

Figure 3. Columbia Mental Maturity scores for normal, left, right, and bilaterally brain-damaged subjects.

Figure 4. Raven's Coloured Progressive Matrices scores for normal, left, right, and bilaterally brain-damaged subjects.
Performance data for the Hanoi Towers puzzle did not meet criteria required for statistical analyses. The problem with these data analyses centers around the subjects that terminated their effort and thus did not achieve problem solution. For example, how does one compare the subject that quit trying to solve the problem after 5 moves with the subject that solved the problem but required 23 moves?

We elected to attempt to provide some descriptive analyses of the Hanoi Towers puzzle data. The amount of time each subject devoted to the problem was calculated. Figure 5 summarizes these data. The mean in seconds for the normal subjects was 111.5. The same figure was 244.5 seconds for the left-brain-damaged subjects, 191.1 seconds for the right-brain-damaged subjects, and 188.1 seconds for the bilaterally-brain-damaged subjects. This indicates that the left-brain-damaged subjects, on the average, devoted more time to the puzzle than did the other subject groups. Normal subjects took the shortest time, followed by the bilaterally-brain-damaged subjects, the right-brain-damaged subjects, and, finally, the left-brain-damaged subjects.

![Figure 5. Mean number of seconds spent on the Towers of Hanoi puzzle by normal, left, right, and bilaterally brain-damaged subjects.](image)

Next the number of moves applied to the problem was examined. All of the normal subjects achieved problem solution ($\bar{X} = 9.1$ moves). This is higher (more moves) than the results obtained by Prescott, Loverso, and Selinger (1984) ($\bar{X} = 7.67$ moves for their normal subjects). Thirty percent of left-brain-damaged subjects did not achieve problem solution and terminated the task. The left-brain-damaged patients studied by Prescott, Loverso, and Selinger (1984) required a mean of 14.67 moves. Comparison of the groups is difficult in that Prescott, Loverso, and Selinger added 20 moves for each subject who gave up the task. When a similar calculation was made for the present left-brain-damaged subjects, a mean of 13.2 moves
was obtained, a score close to the 14.67 moves in the 1984 study. Eighty percent of the right-brain-damaged group and 60 percent of the bilaterally-brain-damaged group completed the task or achieved problem solution. The mean number of moves required was 17.5 moves for the right-brain-damaged group and 18.4 moves for the bilaterally-brain-damaged group. The normal group was considerably better at solving the problem than were the brain damaged groups, while those with brain damage had the following mean scores when 20 moves were added for noncompletion: left = 13.2 moves, right = 17.5 moves and bilateral = 18.4 moves.

To determine the relationships among the four measures administered to each subject, Pearson Product Moment Correlation coefficients were calculated for each possible pair for each group studied. Table 1 presents the obtained coefficients for each comparison. It can be seen that for normal subjects some predictable relationship existed between PICA scores and the CMMS (r = .80) as well as between Raven's performance and CMMS performance (r = .83).

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For the left- and right-brain-damaged subjects there did not appear to be any very meaningful relationships among any of the variables tested. For the bilaterally-brain-damaged subjects the data were somewhat similar to that obtained for the normal subjects. When all groups (scores) were combined the
Raven's score appeared to be the best predictor of the CMMS score, with an obtained correlation coefficient of .72.

For the left-brain-damaged group, performance did not appear to be related to severity or locus of lesion. This finding was similar to that reported by Prescott, Loverso and Selinger (1985).

An analysis was made of possible differences that may have occurred within the bilaterally-brain-damaged group. The 10 subjects within this group were 5 closed-head-injured patients and 5 with non-closed-head etiologies. A repeated measures analysis of variance (1984) yielded no significant difference between the groups (F < 1) for any of the measures: PICA, Raven, CMMS, and completion time for the Hanoi Towers puzzle.

In conclusion, for our subjects none of the measures (with the exception of the PICA) appeared to provide much differentiation between the groups. The PICA overall score differentiated between all of the groups, including the normal group. Normal subjects differed from the other groups on the CMMS but that measure did not differentiate among the pathologic groups. Raven's Coloured Progressive Matrices failed to differentiate any of the groups from each other.

One might ask what these nonverbal tests measure. The Hanoi Towers number of moves appears not to differentiate between pathologic and normal groups in that a large overlap of scores exists between these groups. However, time required to complete the puzzle does separate normal from brain-damaged subjects.

This experiment raises questions regarding what is being measured by nonverbal problem-solving tasks for normal and brain-damaged subjects. Prescott, Loverso, and Selinger (1984) speculated that problem-solving abilities should be studied further. Such work should be geared toward determining the effects that problem-solving deficits have on language functioning. Speculation in the literature (Chapin 1981) suggests that such problems may in fact be related to language functioning.

Our data support the notion that, at least for these types of tasks, reduced ability is a function of brain damage and not of the locus of that damage. Lower level, perhaps nonverbal, problem-solving tasks may have equal potential. This idea is supported by Brown (1983) and by Perelman (1983) who stated that... "preliminary cognitive processes are mediated by brain levels shared by right and left hemispheres, whereas end stage processing is mediated by the left hemisphere." (p. 4) The language deficit still appears to be number one in aphasia.

Finally, discussion following our 1984 paper suggested trying to determine whether or not subjects had any awareness of how they had solved the Hanoi Towers puzzle. Our subjects, regardless of group, did not. Most said something like, "I don't know. I just moved them."

REFERENCES


**DISCUSSION**

Q: Did you do any constructional abilities tests in these patients and look at any relationship there?
A: No. It would be a good thing to look at, but we didn't do it. The patients were, however, able to perform the motor task.

Q: Some years ago, I did some work with the Columbia Mental Maturity Scale. I was wondering if you had analyzed your Columbia data in terms of abstract versus concrete items, and whether you analyzed the paired versus the single items.
A: No. All we analyzed was overall score or number of correct responses.

Q: Well, I suggest a further analysis using these variables.
A: That may tease out some of these variables, but all we were interested in was an overall nonverbal problem solving task to see whether or not it looked as if one group was different from other groups. For the different measures that we had it appeared that they weren't, and also that the different measures weren't tremendously successful, at least for our patients, in teasing out or identifying differences between groups.

Q: How would you use this clinically in treatment?
A: I think what we concluded was not that problem solving wasn't important or that it may not be important, but the quote from Perecman suggests that there are different levels of functioning and that when you make things more linguistic maybe then you move into levels that would result

-256-
in differentiation based on where the lesion is. For our patients these nonverbal problem-solving tasks didn't really differentiate patients all that well, perhaps because we had so much overlap of people having success. Even people in the pathologic group had success. That didn't tell us very much. What it says to me is that, we should look clinically at problem solving and that may tell us a lot about treating our patients.

Q: What you found is that it doesn't matter whether one's damage is in the right hemisphere, or the left hemisphere or both on this Hanoi Towers puzzle. There were no differences in those three lesioned groups. This kind of task, I think, should be sensitive to intrahemispheric lesions and that frontal, especially prefrontal lobe lesions, would have inordinate difficulty compared with more posterior lesions.

A: Based on things like fluency, we didn't see differences in Hanoi Towers performance between posterior or anterior lesions.

Q: Luria talked about patients with prefrontal lesions who were unable to verbally mediate performance. He noticed some people that would verbalize the solution, but make the wrong moves motorically.

A: We didn't see anyone verbalize that way. We also tried to get at that by asking the subjects how they solved the problem. Last time (1984) you asked if people could tell you how they solved the problem and not only could the pathological groups not tell you how, but the normal subjects could not tell you either.

Q: If you get your subjects very early on, and looked at them longitudinally, early on might you find that for many subjects their performance would relate very highly to the degree of their aphasia and that over time, their performance would not be so closely related to severity? I'm wondering whether initially these people were verbally-mediated problem solvers and over time, in reaction to their language deficits, they might move to less verbally-mediated problem solving and therefore exhibit less relationship to their aphasia.

A: We didn't look at that, but we can speculate a little about that from the last paper. Every subject on our first study was six months post onset. This time we had people at a variety of times post onset, I can't tell you right now how many were in each time slot, but the results on the Hanoi Towers puzzle tended to be pretty similar for both studies. Then if you look at severity, there seem to be people at the bottom half of the subject group who took a lot more moves to solve the problem and there are also people in that severity range that solved the problem in seven moves. The same is true for the high end (least severe) of the continuum.

Q: Do you think that it's possible that the variability you see could be related to their premorbid problem solving style?

A: I don't think so. The other measure, the PICA, showed that variability.

Q: I noticed that on a couple of your measures that your bilateral people were between the rights and the lefts and on the surface that would make me think, that at least for these kinds of tasks, it's better to have bilateral damage than either right or left damage. It probably has something to do with the severity of the patient group. How do you
go about matching severity if you think it is an important variable? Do you do it by lesion volume, or on a nonverbal basis because the right hemisphere patients have a nonverbal problem?

A: Well, I don't know how you do it. Many people here are trying to answer that question by saying these are fluent, these are nonfluent, these are posterior, these are anterior, and talking about differences and trying to relate it to size of lesion or site of lesion. It appears to me that these measures aren't the best way to do it, and that's what I think we learned from this study. These measures are not sensitive to these kinds of differences, because groups and subjects didn't perform differently.

Q: Some of the work we've been doing on discourse analysis over the past years includes a lot of cognitive testing. Although we haven't done Hanoi Towers, we've done picture arrangement, block design, and Raven's and one of the strongest relationships we've found with the ability to produce discourse is ability on block design. That is with left hemisphere patients. We haven't done rights. So I think that's a real possibility that patients' abilities on nonverbal problem solving tasks will reflect some of their higher-level linguistic abilities as measured by discourse.

Q: Did the right-hemisphere patients react any differently to the task? It's been found that right-hemisphere patients resist novel and unfamiliar tasks.

A: No, they participated in similar fashion as the other groups.

Q: You said you found no difference between rights, lefts, and bilaterals, but, perhaps if the groups had been equated on severity, you might have found a difference. Maybe rights who are as severe as lefts do perform differently or worse. We do a lot of studies where we compare right to left to bilateral, and I don't think anybody's yet found a measure that you can equate them on. Most studies just assume there is no selective bias by just taking all of their left, right, and bilateral patients. But one study that's been done looking at size of the lesion on C.T. has found a significant effect of severity. It may be that we throw out more of the severe rights because they are untestable or because of neglect or refusal on the test.

A: I agree. If we had some mechanism for equating severity those equated groups might not have been equal. A design issue for dealing with that problem is to include everyone and don't exclude anyone, or to take a random sample and supposedly those factors cancel each other out. Maybe they just cancelled each other out, so there was no difference.

Q: Did your bilateral group have any traumatic head injured patients and how many?

A: We had five closed head injured, and five who weren't, and there was no difference between the two groups that we found.

Q: I've found that if I'm working with scrambled sentences with right-hemisphere patients, that if I let them talk, and if they read the scrambled words they put the scrambled sentences together right away.
If you don't let them talk, it takes an inordinate amount of time to come out with the sentence. So, I'm wondering how some of these patients might do if they were encouraged to talk.

A: We didn't restrict them from talking, or encourage them to talk.